Tutorial 3: Parallel Flows & Spring Washer

In the previous tutorial we added transmission to the model, which involved introducing the power flow constraint. Now we will investigate the impact that the power flow constraint has on a model with parallel branches, which will lead on to mesh flows, and the spring washer effect.

Parallel Flows

Unconstrained parallel flows

Build and solve the parallel branch model shown in Figure 104 by tapping Bus-Bus-Gen-Load-Branch-Branch and then Solve with the default values and all solve settings selected to OFF.

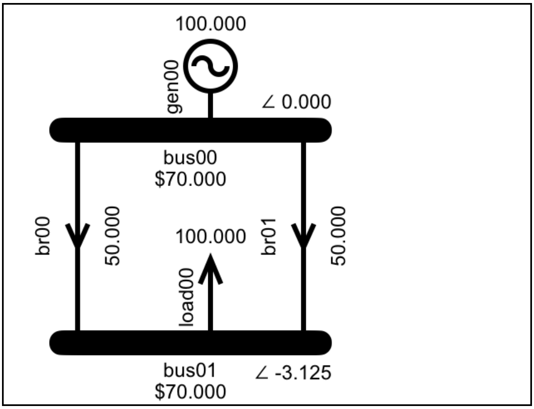


Figure 104: Parallel branches model

As described in the previous section, the power flow constraint calculates branch flow as the product of the branch susceptance and the phase angle difference between the from-bus and the to-bus. Therefore, because both branches in this example have the same susceptance and are connected to the same buses, their power flow is the same.

Parallel flows with a binding branch

Parallel branches get more interesting when one of the branches is binding. To see this, reduce the maximum flow limit on br00 from 300MW to 40MW. The result is shown in Figure 105.

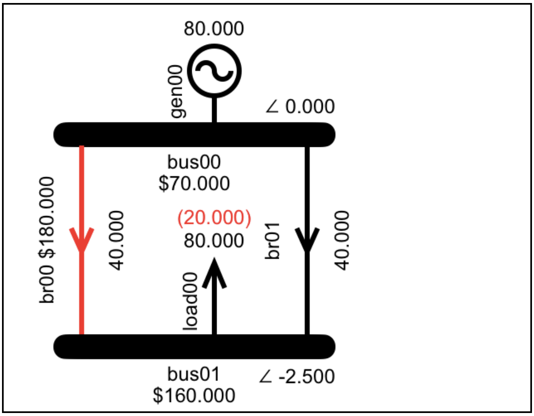


Figure 105: Binding branch restricts flow on parallel branches

We see that although the maximum flow on br01 remained at 300MW, its scheduled flow is now 40MW. The explanation is the same as for the unconstrained case… both branches have the same susceptance and are connected to the same buses; hence their scheduled flow must be the same. In this case br00 is binding on its maximum flow of 40MW; the branch flows must still be the same, but they must both be lower.

Capacity cost

When br00 was binding and there was no parallel branch (see the binding branch example in the previous tutorial), the branch had a capacity cost of $90/MW. Now it is double that because an incremental increase in flow on br00 will also allow a corresponding increase on br01.

Parallel flows with different susceptance values

The previous example shows that parallel branches with the same susceptance and the same end points must have the same flow. Not unexpectedly, if parallel branches have different susceptance values then they will have different flows.

To see this, edit br00 to double its susceptance from -16 to -32. The flow on br00 will be twice that on br01. Branch br00 will still bind, but this time the flow on br01 will be half of br00, as shown in Figure 106.

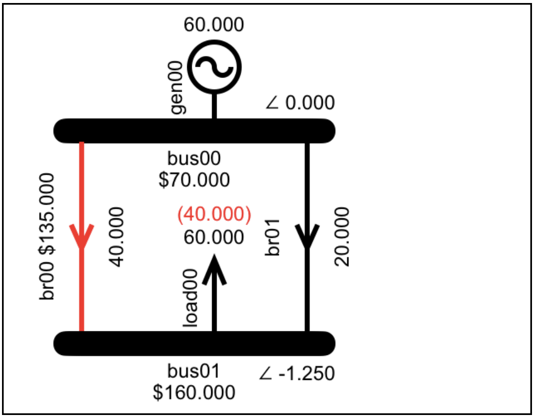


Figure 106: Branch br00 susceptance twice that of br01

Conversely you can change the branch parameters to see what happens when the susceptance of the non-binding branch br01 is increased. This will result in br01 transmitting more of the power; if the br01 susceptance is increased enough then it will take enough of the flow so that br00 will no longer bind.

Susceptance and reactance

To investigate the impact of different susceptance values we could also change the reactance, because susceptance is calculated from reactance and resistance, as seen in Equation 7.

Changing the susceptance as we have been doing provides a more direct demonstration because it is the susceptance value that is used in the constraint, e.g., halving the susceptance will halve the flow, whereas doubling the reactance will not have exactly the same effect because the reactance is combined with the resistance to calculate the susceptance. You can check for yourself by changing the reactance and seeing what happens.

The Spring Washer effect

Building and solving a three-bus model

Now that we have seen how a binding branch can limit the flow on parallel branches, we will take this a step further and look at what happens when there are multiple paths between the generation and the load. This model is then used to investigate and explain the spring washer effect.

To create multiple paths we need at least a three-bus model. Create this by adding three buses (tap Bus-Bus-Bus), add a generator and a load (tap Gen-Load), and then add two branches (tap Branch-Branch). This will produce the (incomplete) model shown in Figure 107.

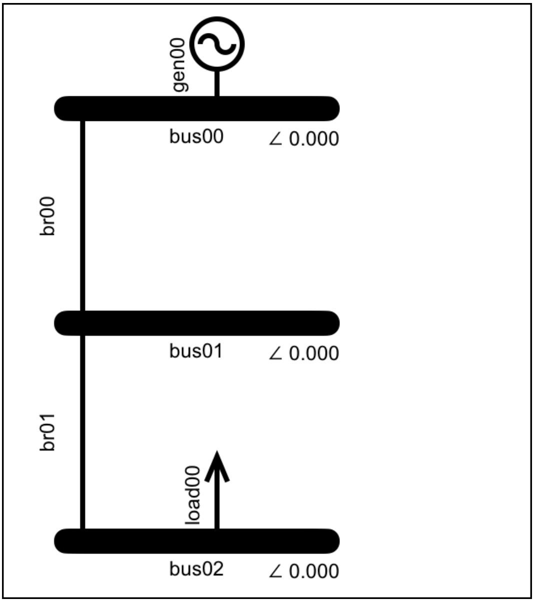


Figure 107: Building the three-bus model, step 1

To make adding the third bus as easy as possible, first resize bus00 and bus02 by dragging them from their right hand ends, so that they are the length shown in Figure 108. Then add another branch, which will automatically connect to the end of the two buses that have been re-sized.

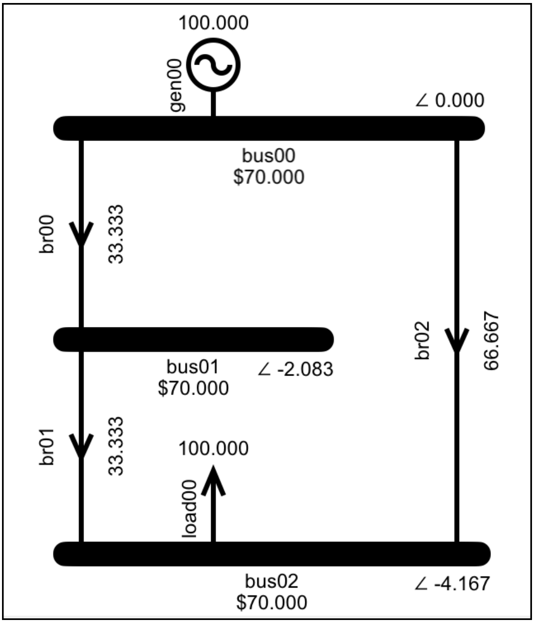


Figure 108: Three bus model

Solving the model with all Solve Settings selected OFF produces the result shown in Figure 108. The branches have the same susceptance. Therefore, at first glance, it may seem that they should all have the same flow but they don't because they are not all connected to the same buses.

The branches cannot have the same flow because for br00 and br02 to have the same flow the power flow constraint would require them to have the same phase angle difference, which would require bus01 to have the same phase angle as bus02. But that would result in a zero phase angle difference across br01, which would only work if br01 flow was zero, which it can’t be because the node balance constraint requires that the flow into bus01 must match the flow out.

The result shown in Figure 108 meets the requirements of the constraints; *due to the way that the power is flowing*, br00 and br01 are in parallel with br02, hence the sum of their phase angle difference must be the same as the phase angle difference across br02. Because all of the branches are the same, the phase angle difference across each of br00 and br01 must be half that across br02, and therefore each must have half the flow of br02.

Another way of looking at it is that the impedance across br00 and br01 is twice that across br02, hence the flow down that path is halved (mathematically the impedance of series conductors is additive, whereas the overall susceptance reduces).

Losses were not included and no branches are binding; hence all prices are the same.

This example shows how a meshed network affects the distribution of the power flow. This result is the pre-cursor to explaining the spring washer effect.

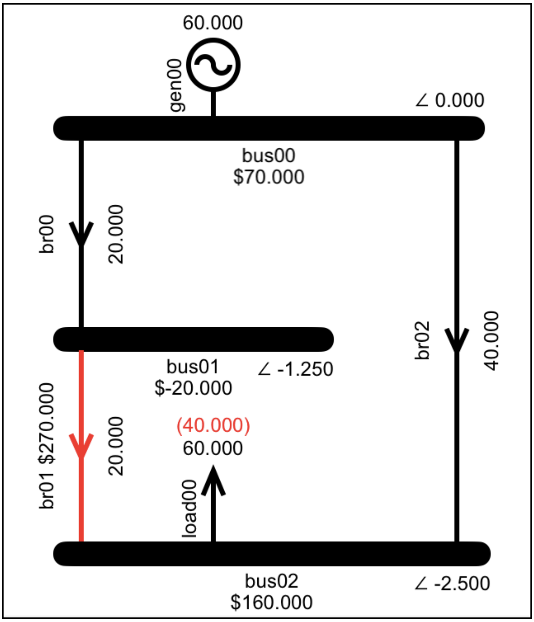


Figure 109: The spring washer effect

Creating the spring washer effect

Create the spring washer effect by double tapping br01 and lowering its maximum flow from 300MW to 20MW; currently it is scheduled at 33.33MW, so reducing its maximum to 20MW will cause it to bind. Solving this model produces the result shown in Figure 109.

Why it is called the spring washer effect

The “spring washer” in the spring washer effect refers to the similarity between the “shape” of the prices shown in Figure 109 and the shape of a spring washer, shown schematically in Figure 110.

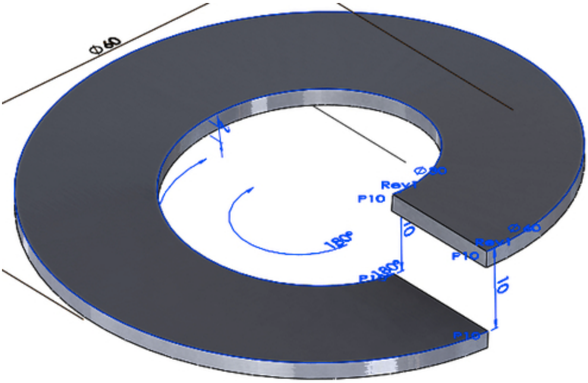


Figure 110: Representation of an actual spring washer

The gap between the highest and lowest points of the actual spring washer are represented by the gap between the highest and lowest prices either side of the binding branch; the prices either side of the branch are not financially connected by the branch, even though the branch physically connects the buses.

The curve of the spring washer is represented by the prices passing from the lowest price at bus01 on the upstream side of the binding branch, through the medium price of $70 at the generator bus (bus00), then via br02 to the highest price of $160 at the load bus (bus02).

Explaining the positive prices

The prices at bus00 and bus02 are as explained in Tutorial 2: Modelling Transmission, i.e., the uncleared bids at bus02 set the bus price at bus02 because the value of electricity at this bus is the value of the bids that could be cleared. The cleared offers at bus00 set the price at bus00 because the value of extra electricity at this bus is the value caused by not having to clear the existing offers.

Explaining the negative price

The explanation of any bus price is that it indicates the $/MW rate of change to the objective value due to relaxing the node balance constraint at that bus.

For the spring-washer example we need to expand on this slightly. A positive bus price indicates that an increase in available power will increase the objective value, while a negative bus price indicates that an increase in available power will decrease the objective value. These statements are also true if both cause and effect are reversed; a positive bus price indicates that a decrease in available power will decrease the objective value, while a negative bus price indicates that a decrease in available power at the bus will increase the objective value.

As before, we can investigate the price at bus01 by seeing what happens when its node balance constraint is relaxed. In previous examples we relaxed the node balance constraint by adding a 1MW offer at $0/MWh. In the spring washer example we are looking at now, a $0/MWh generation offer will not clear at bus01 because the negative price indicates that there is a disincentive to making power available at bus01.

The disincentive of extra power

For an offer to clear at a bus, the *cost* of its power must be less than or equal to the *benefit* of extra power at that bus. The benefit of extra power at bus01 is -$20, therefore only an offer with a cost less than or equal to -$20 will clear, i.e., a generator that is prepared to *pay* $20/MWh to generate.

Figure 111 shows the result of adding a generator to bus01 with an offer of 1MW at -$20/MW. The offer clears and we can use the result to explain how the extra 1MW at bus01 impacts the objective value.

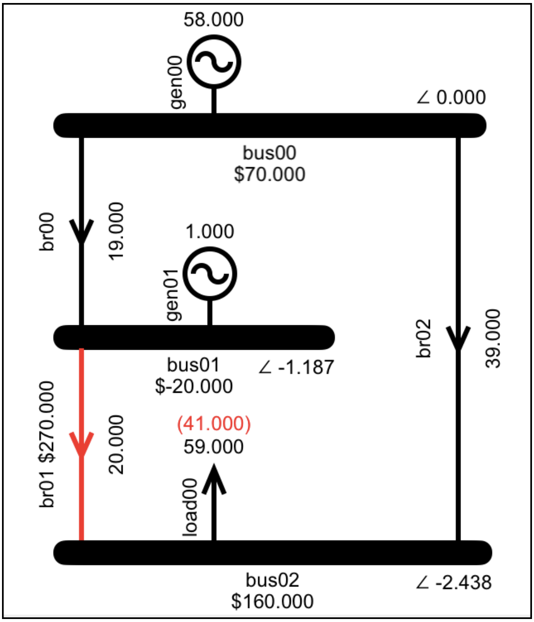


Figure 111: Impact of an extra 1MW at bus01

The changes in the results due to relaxing the node balance constraint at bus01 are explained as follows:

The extra 1MW at bus01 heads toward the load via br01; hence the existing inflow to bus01 via br00 must be reduced by 1MW to keep br01 on its limit.

To reduce the inflow to bus01 from br00 the phase angle at bus01 is reduced. Because br01 is connected to bus01, in order to keep the flow on br01 at its limit the phase angle at bus02 must also be reduced.

Reducing the phase angle on bus02 forces the flow on br02 to reduce by 1MW. The direct impact of this is that the cleared bid quantity at bus02 must reduce by 1MW.

Overall the flow out of bus00 was reduced by 1MW on br00 and 1MW on br02; therefore the cleared offers at bus00 must reduce by 2MW.

Table 5: Change in objective for 1MW extra at bus01

Component	△Quantity	Price	△Objective
gen00	-2MW	$70	+$140
load00	-1MW	$160	-$160
objective			-$20

The impact on the objective value of the 1MW of extra power at bus01 is detailed in Table 5. The calculated change in the objective value is $-20; this lines up with the bus price at bus01.

Note that if bus01 had been the reference bus then it would have been the phase angle at bus00 that would have been reduced… the overall effect would have been the same, as shown in Figure 112.

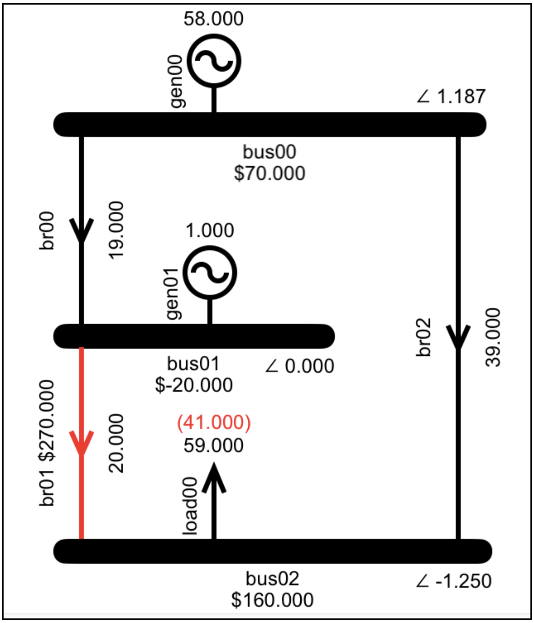


Figure 112: Overall the same result with bus01 as reference

No change in objective value

The difference between this experiment to explain the objective value and those conducted previously is that the calculated change in the objective value presented in Table 5 is not reflected by the change in the actual objective value. This is because the price of the offer that relaxed the node balance constraint is not $0; we had to make it $-20 so that it would clear. Therefore, even though the *calculated* impact on the objective value was $-20, this is cancelled out by the negative cost, i.e., the benefit, of clearing the offer with the -$20/MWh price.

The -$20 bus price indicates that generation must *pay* at least $20/MWh in order to be cleared at bus01. It also indicates that load will be *paid* $20/MWh if it is situated at bus01. We can see this by deleting the generator at bus01 and adding a load.

Relaxing node balance by adding load

The negative price at bus01 indicates that more power flowing into the bus will make the objective value worse. This also indicates that more power leaving the bus will make the objective value better; we can demonstrate this by adding load at bus01.

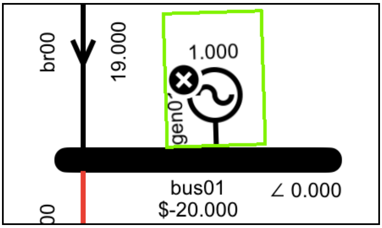


Figure 113: Press and hold gen01 to initiate the delete, tap gen01 to delete, tap anywhere else to cancel

First we need to delete the extra generator that we added. Delete gen01 by pressing it until it starts to wobble and a cross appears, as shown in Figure

113. Then tap gen01 to perform the delete, or tap anywhere else to cancel.

Now add a load to bus01 and edit it so that it has a bid of 1MW at $0. Solve, and the result is as shown in Figure 114.

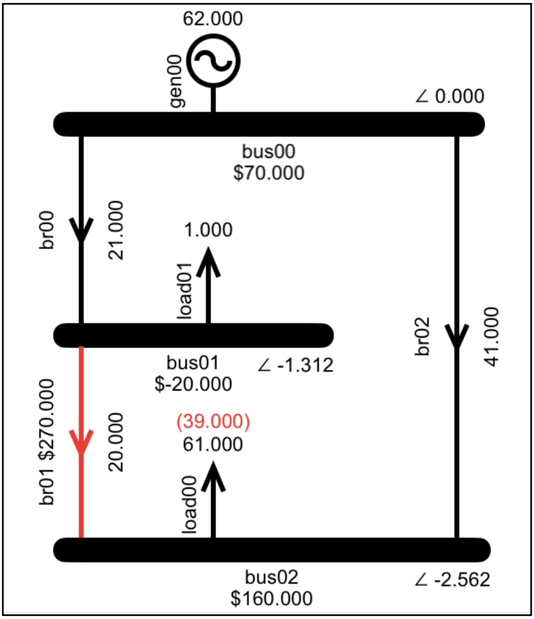


Figure 114: Spring washer with node balance relaxed by adding load

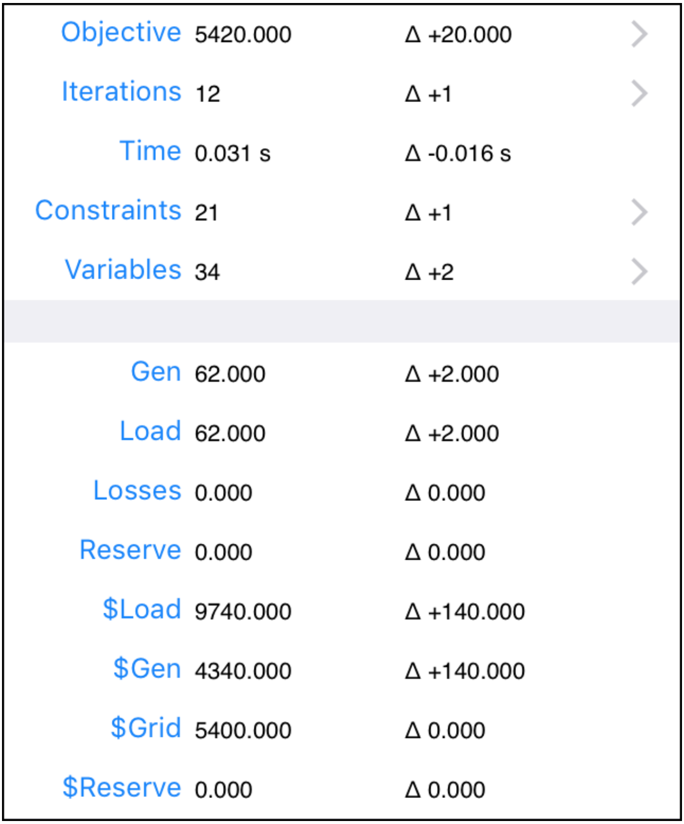
The explanation of the result when load is added is generally similar to the generation example above; power is supplied to the extra 1MW of load by increasing the flow on br00. This increased flow requires an increase in the phase angle at bus01, which in turn requires an increase to the phase angle at bus02, to keep br01 on its limit. The increase in the bus02 phase angle forces more flow along br02, which allows more of the bids to clear at bus02. Overall, increasing the load at bus01 provides a benefit, by way of the benefit due to increasing cleared bids at bus02, which outweighs the cost of increasing the cleared offers; hence the extra load at bus01 has improved the objective value.

Compared to the original result shown in Figure 109, the impact that the extra 1MW of load has on the objective value is calculated in Table 6.

Table 6: Change in objective for 1MW load at bus01

Component	△Quantity	Price	△Objective
gen00	+2MW	$70	-$140
load00	+1MW	$160	+$160
load01	+1MW	$0	$0
objective			+$20

This is confirmed by the Results in Figure 115 showing that the extra 1MW of load resulted in a $20 increase in the objective value. Because this increase was due to *less* power at the bus, this translates to a bus price of -$20.



Objective	5420.000	Δ +20.000
Iterations	12	Δ +1
Time	0.031 s	Δ -0.016 s
Constraints	21	Δ +1
Variables	34	Δ +2
Gen	62.000	Δ +2.000
Load	62.000	Δ +2.000
Losses	0.000	Δ 0.000
Reserve	0.000	Δ 0.000
$Load	9740.000	Δ +140.000
$Gen	4340.000	Δ +140.000
$Grid	5400.000	Δ 0.000
$Reserve	0.000	Δ 0.000

Figure 115: Node balance constraint relaxed by 1MW of load improves objective by $20

Spring washer prices not always negative

The characteristic of the spring washer effect is that the bus with the lowest price, i.e., the bus on the upstream (generator) side of the binding constraint, will have a price that is lower than the bus prices either side of it… but it is not necessarily a negative price.

We can demonstrate this by increasing the offer price at gen00 from \$70/MWh to \$100/MWh, which produces the result shown in Figure 116. The price at bus01 is still the lowest, but is no longer negative.

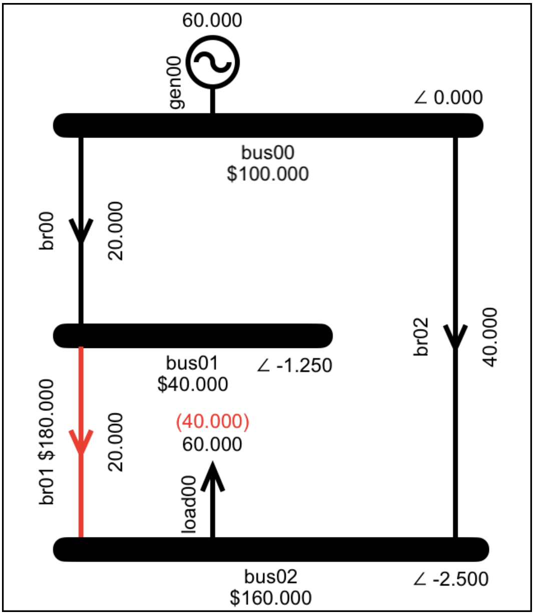


Figure 116: Spring washer price not negative, due to higher offer price

Also, returning gen00 to its original offer price of \$70/MWh but lowering the susceptance of br02 from -16 to -5, we see that by making br02 less attractive to power flow the benefit of load at bus01 decreases, producing the result shown in Figure 117.

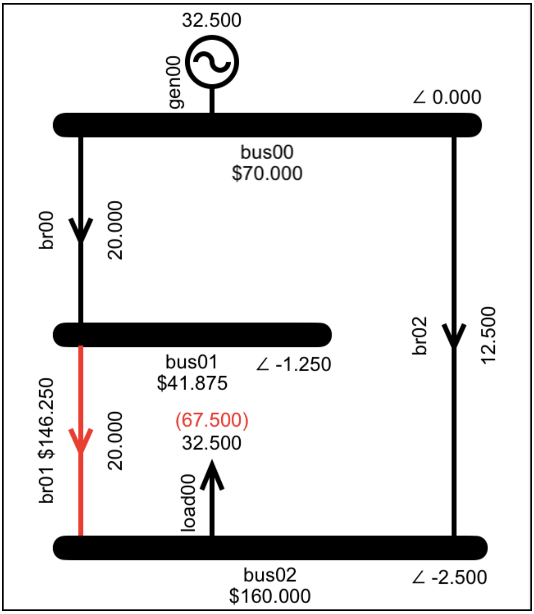


Figure 117: Spring washer price not negative, due to lower susceptance on br02

Summary

The examples in this section showed that because the power flow constraint links branch flow to bus angle, a branch's flow may be restricted due to limits that are binding on parallel branches.

We also saw that the impact of parallel branches can result in the spring washer effect, whereby the price at a bus situated between the generation and the load can end up with a negative price, indicating that non-paying load at that bus will improve the objective value. We saw that this is because load at the bus will influence the phase angle difference across the binding branch, thereby enabling increased flow on branches parallel to the binding constraint allowing more paying load to clear.