Tutorial 1: Explaining Prices

This tutorial explains how the electricity market is represented as a Linear Programming (LP) mathematical model and how solving the LP model produces the electricity prices.

Default parameters and settings

Most of the parameters used in the following tutorials are the default values, as shown in Figure 77, Figure 78 and Figure 79. If you have set other default values and want to reset to the originals then you can use the "Reset Default Values" button on the Settings display.

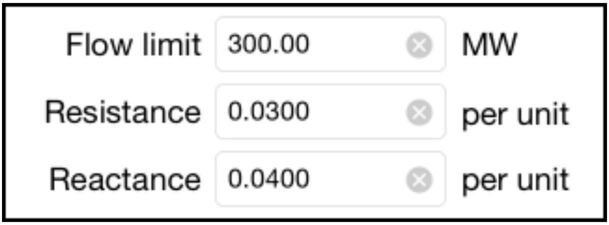

Figure 77: Default branch parameters

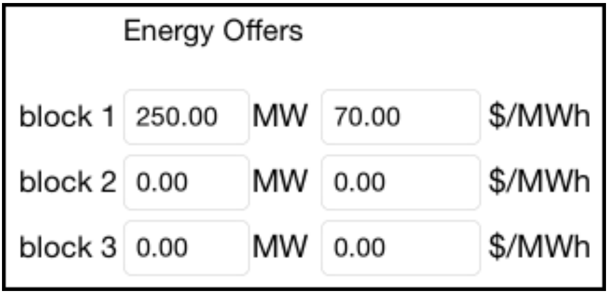

Figure 78: Default energy offers

Figure 79: Default load bids

Unless otherwise stated, the Solve Settings used by the worked examples are assumed to be those shown in Figure 80; obviously when line losses are examined the instructions will be to set Include Losses to ON, and so on for the other examples, but Figure 80 is our starting point.

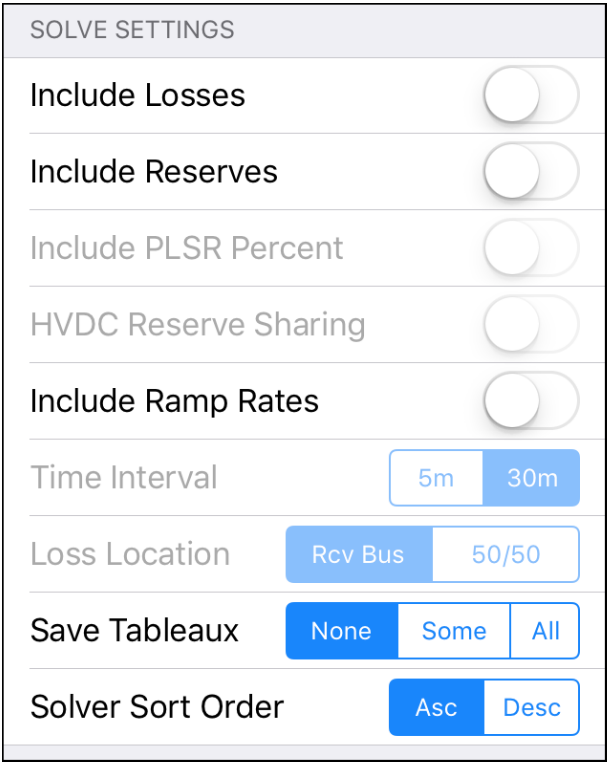

Figure 80: Solve Settings for tutorial 1

Build the model

Build the electricity market model for tutorial 1 by tapping the following buttons on the Build menu: Bus-Gen-Load.

Leaving the default parameters in place, solve the model by tapping the Solve button, checking that all solve options are selected OFF, then tapping the Solve Now button. The software builds a mathematical model of your electricity market and then solves that model using the simplex algorithm. The resulting prices and quantities are displayed in Figure 81.

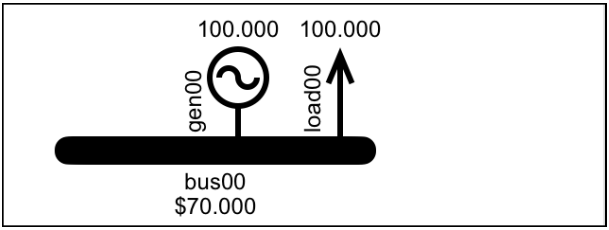

Figure 81: Small electricity market

Linear programming

The mathematical model of the electricity market is a Linear Programming model, referred to as an LP model.

The following excerpt from http://www-history.mcs.st-and.ac.uk/~history/Biographies/Dantzig_George.html explains the term “Linear Programming”:

In 1947 George Dantzig made the contribution to mathematics for which he is most famous, the simplex method of optimisation. It grew out of his work with the U.S. Air Force where he become an expert on planning methods solved with desk calculators. In fact this was known as "programming", a military term that, at that time, referred to plans or schedules for training, logistical supply or deployment of men.

Dantzig mechanised the planning process by introducing "programming in a linear structure", where "programming" has the military meaning explained above.

Objective function and constraints

The general form of an LP model consists of an equation that defines the objective of the model and a series of equation constraints that represent the behaviour of the system being modelled. As the simplex algorithm pursues the objective, it must ensure that the requirements of the constraints are met. This concept is illustrated in Figure 82.

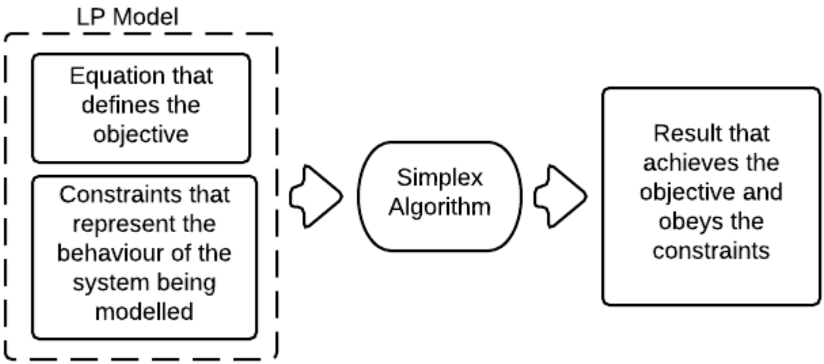

Figure 82: LP Model, solved using the Simplex Algorithm

Bus variables and constraints

The constraints that model the behaviour of the electricity market are associated with the components of the model, i.e., the buses, branches, generators and loads.

To view the constraints for bus00, double-tap bus00 to view its Data Display, then on the Data Display tap the ∑ button indicated in Figure 83. This will take you to the variables and constraints display shown in Figure 84.

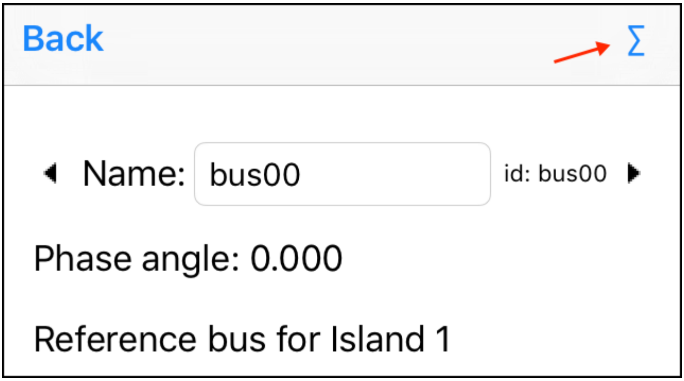

Figure 83: Data Display for bus00 with the "variables and constraints" button indicated

Figure 84: Variables and constraints display for bus00

Node balance constraint

In this single-bus model there are no variables associated with bus00. However there is a constraint, which is the node balance constraint whose general form is shown in Equation 1. This constraint models the physical reality that the power flowing into the bus must equal the power flowing out.

$$\sum Bus_{FlowIn} - \sum Bus_{FlowOut} = 0$$

Equation 1: Node balance constraint

When we clicked the “Solve Now” button the software created constraints to represent the physical reality of the model. Figure 84 shows that the implementation of the node balance constraints ensures that the cleared generation of gen00 matches the cleared bids of load00.

The constraint presented in Equation 1 is an equality constraint (i.e., *a* equals *b*), but the simplex algorithm requires that all constraints are expressed as <= equations. Hence, the node balance constraint is expressed as a combination of <= and >= constraints, which together have the same effect as a single equality constraint… then the >= constraint has its signs reversed, to become a <= constraint.

Bid and Offer constraints

Bids and offers consist of a quantity and a price. The portion of the quantity that is scheduled is the cleared quantity. A constraint is required to limit the cleared quantity to be no more than the bid or offered quantity. These constraints are shown in Equation 2 and Equation 3.

$$ClearedQuantity_{LoadBid} \leq MaxQuantity_{LoadBid}$$

Equation 2: Cleared bid constraint, i.e., bid max constraint

$$ClearedQuantity_{GenOffer} \leq MaxQuantity_{GenOffer}$$

Equation 3: Cleared offer constraint, i.e., offer max constraint

You can view these constraints via the "Variables and Constraints" displays for load00 and gen00 as shown in Figure 85 and Figure 86. As with the bus, these displays are accessed by double tapping the component to access the Data Display, then tapping the ∑ button in the toolbar.

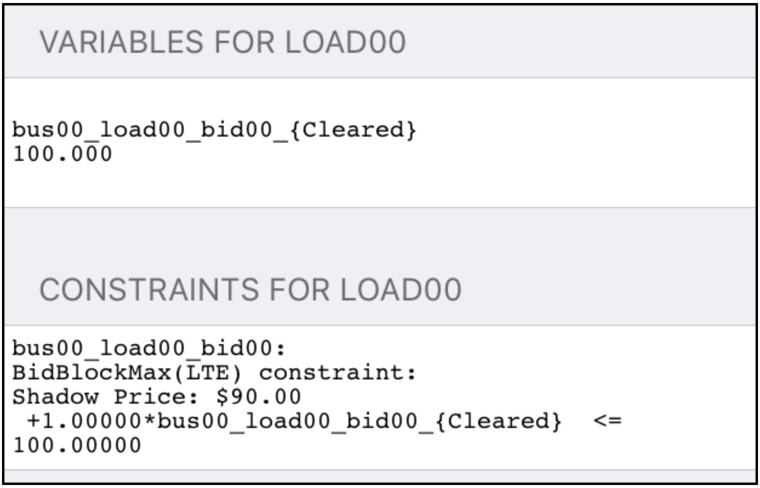

Figure 85: Variables and constraints for load00

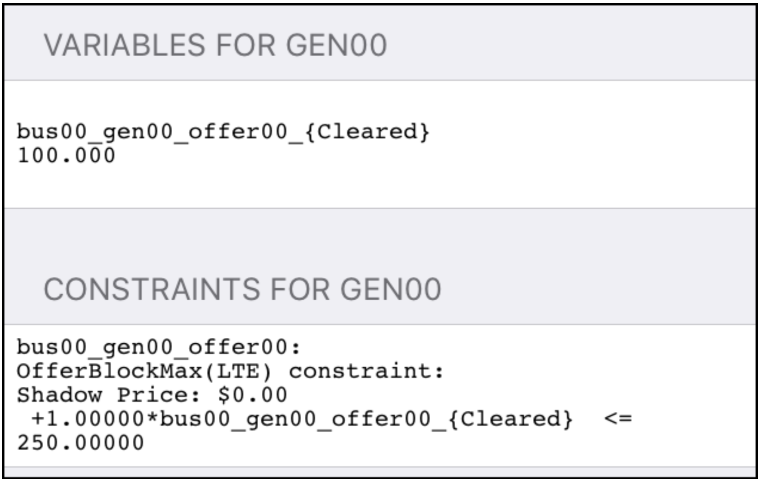

Figure 86: Variables and constraints for gen00

The Objective Function

The LP model consists of constraints representing the behaviour of the system being modelled, and an equation that defines the objective value. Maximising the objective value is what drives the actions of the simplex algorithm.

The equation that defines the objective value is called the objective function. Equation 4 describes the objective function for the electricity market model. This shows that the objective of solving the electricity market model is to maximise the benefit of the cleared bids while minimizing the cost of the cleared offers.

Maximize: $ObjectiveValue$
$$= loadBid_{Cleared} \times loadBid_{Price} - genOffer_{Cleared} \times genOffer_{Price}$$

Equation 4: Objective function for the electricity market model

The objective value for the latest solve is shown on the Results display, along with how much the objective value has changed relative to the previous solve.

To see the details of how the objective value was calculated, tap the Objective row on the Results display, which will take you to the Objective display shown in Figure 87.

Figure 87: The Objective display shows how the objective value is calculated

The electricity price

The bus price of $70/MWh represents the value of electricity at the bus. In terms of the LP model, this bus price represents the change in the objective value that would occur if the node balance constraint for the bus was relaxed. This is explained as follows.

What it means to relax a constraint

If we add 1MW to the node balance constraint, as shown in Equation 5, we have relaxed the node balance constraint by 1MW.

$$Gen_{bus00} - Load_{bus00} + 1\text{MW} = 0$$

Equation 5: Node balance constraint relaxed by 1MW

Relaxing the constraint by 1MW effectively adds 1MW of power to the bus. Observing how the objective value changes due to this additional 1MW will tell us the value of that 1MW, i.e., the value of electricity at the bus.

Relaxing the node balance constraint

We can simulate relaxing the node balance constraint by adding the extra 1MW via a new generator with a $0 offer price.

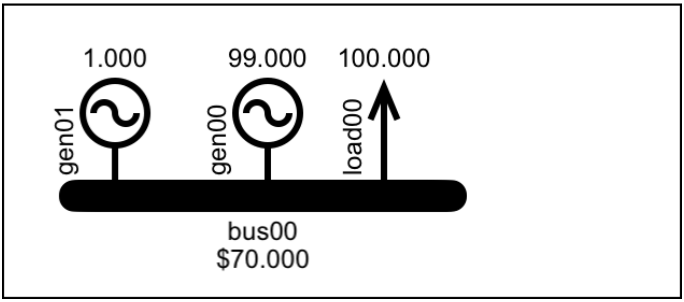

Figure 88: Add new generator, gen01, to the model and drag it into place on the bus

Add the new generator by tapping the Gen button. If necessary drag the new generator into place on the bus, as shown in Figure 88.

Double-tap the new generator to view its Data Display and modify its offer block 1 to be 1MW at $0/MWh as shown in Figure 89.

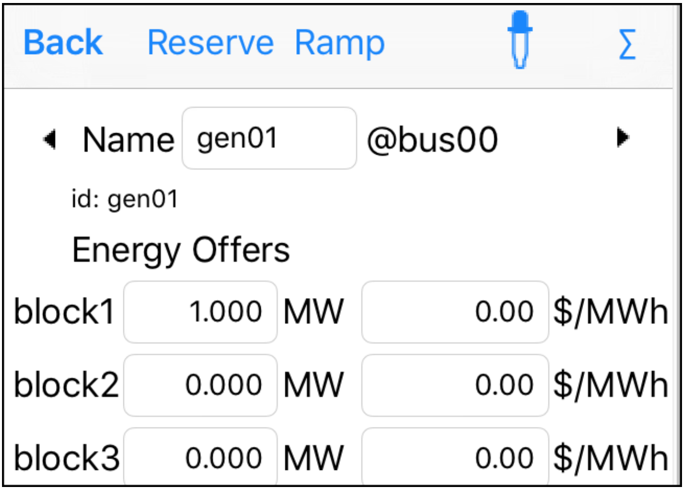

Figure 89: Double-tap gen01 and edit its block 1 offer to be 1MW at $0/MWh

Solve the model, with all solve settings set to OFF. To view what impact this has had on the objective value, tap the Results button. As shown in Figure 90 the objective value has increased by $70.

To achieve this $70 increase in the objective value we relaxed the node balance constraint by 1MW, hence the rate of change of the objective value due to relaxing the node balance constraint for bus00 is $70/MW.

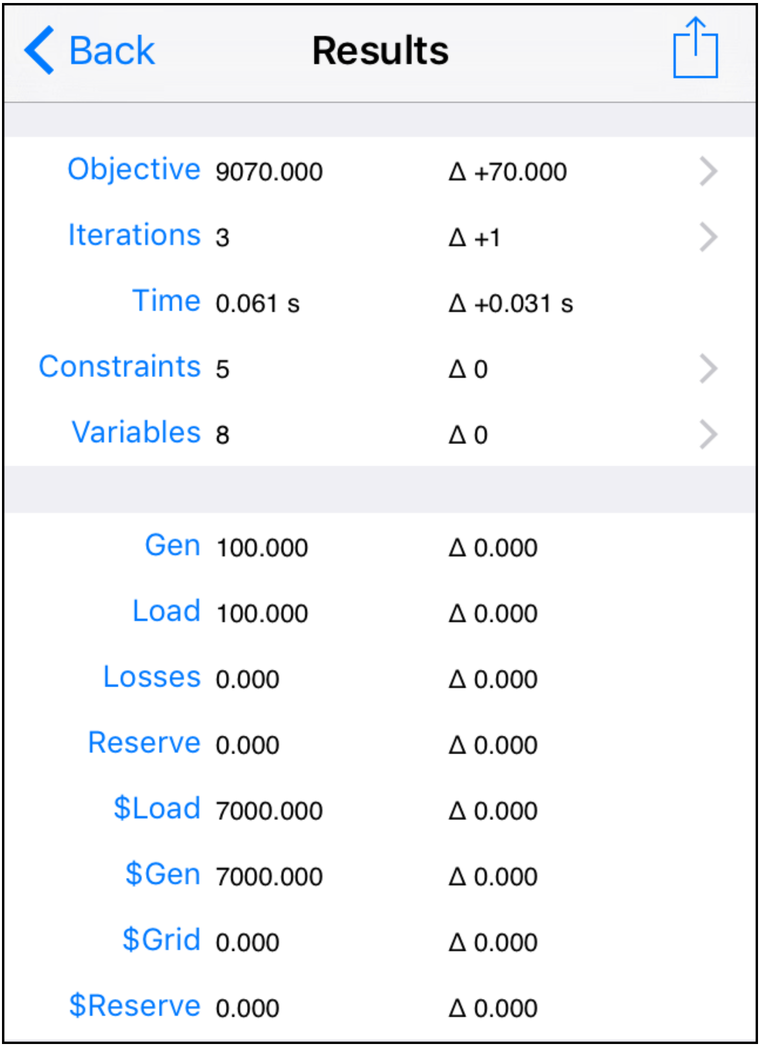

Figure 90: Objective △ $70 for 1MW at $0 → bus price is $70

If we had offered 2MW of generation at $0 then the objective value would have increased by $140. The rate of increase of the objective value, and hence the bus price, would still be $70/MW.

This tells us that the benefit due to generation at the bus is $70/MW. To clear generation with a non-zero offer price the objective value must incur the cost of the offer, which will reduce the objective value. Therefore, provided the offer price is less than $70/MW, overall it will improve the objective value and the offer will clear.

Explaining the bus price

The above example demonstrates that the value of extra electricity at a bus is determined by the impact that it has on the objective value. The example also allows us to explain how this impact occurred, by looking at how the results have changed…

The objective value increased because the additional 1MW at $0/MWh allowed the generation of gen00 to be reduced by 1MW, reducing the overall cost by 1MW x $70/MW = $70, thereby increasing the objective value by $70.

On the Results display, tapping the Objective row takes you to the Objective display shown in Figure

91, which details the calculation of the objective value. Note that because the generation offer that was used to relax the node balance constraint has a cost of $0/MWh, its offer price had no impact on the objective. Hence the change in objective reflects the value of the generation quantity only… it is not influenced by the offer price.

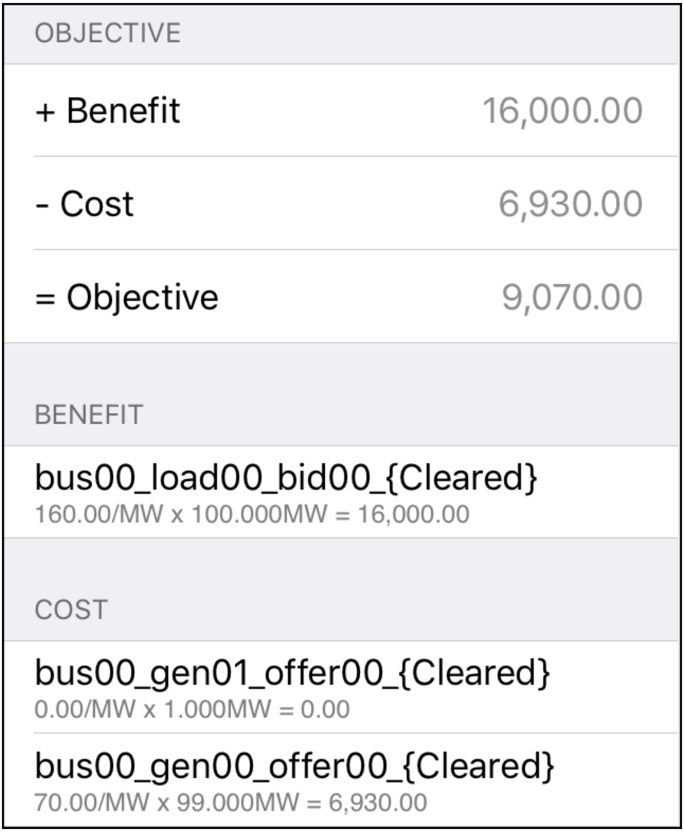

Figure 91: Details of the calculation of the objective value

The shadow price of a constraint

The $/MW improvement in objective value due to relaxing a constraint is referred to as the shadow price of the constraint. The shadow price of a bus's node balance constraint determines the bus's energy price.

The shadow price of *any* constraint in the LP model is the $/MW change in the objective value due to relaxing that constraint. The only shadow price that is immediately useful is the shadow price of the node balance constraint because it sets the electricity price.

You can view the shadow price of any of the constraints by looking on the Constraints display. If a constraint has a non-zero shadow price, then this indicates that the objective value will be changed if the constraint is relaxed. This in turn indicates that the constraint is binding, i.e., the quantity being constrained would have been used more, because it can improve the objective value, if it were not for the constraint.

Summary

In this section we introduced Linear Programming (LP) and demonstrated how a small nodal electricity market is represented as an LP model.

We explained that the shadow price of a constraint is the benefit to the objective value of relaxing that constraint. We saw that the shadow price of the node balance constraint determines the bus price. We also saw how to use the app to relax the node balance constraint and thereby explain the bus price.